Arrangements of Orthogonal Circles with many Intersections

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FernUniversität Hagen

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II. Nonnested Orthogonal Circle Arrangements

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III. General Orthogonal Circle Arrangements

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Introduction

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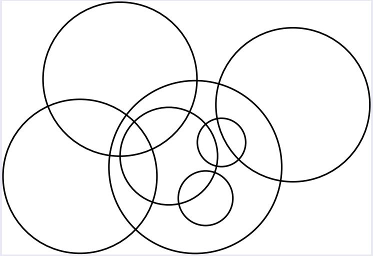
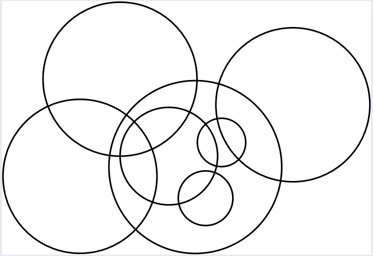


Circle Arrangement

Introduction



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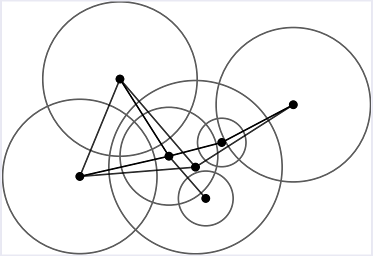
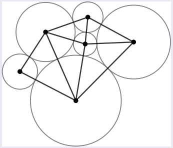


Intersection Graph

Circle Arrangements

Circle Arrangements

- Circle Packing Theorem by Koebe, Andreev and Thurston



Circle Arrangements

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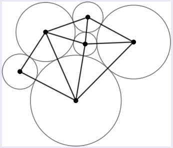
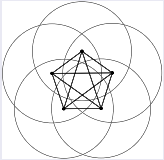
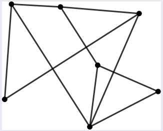


- Overlapping circles are not necessarily planar



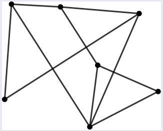
Graph Drawing

- Straight-line RAC-Drawings



Graph Drawing

- Straight-line RAC-Drawings have at most $4n - 10$ edges (Didimo, Eades, Liotta (2011)).



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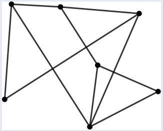
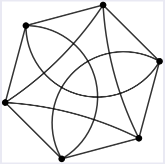


- Arc-RAC Drawings



Graph Drawing

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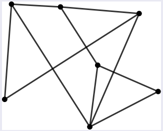
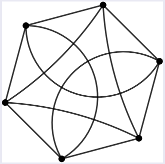


- Arc-RAC Drawings have at most $14n - 12$ edges and there are some with $4.5n - O(\sqrt{n})$ edges (Chaplick, Förster, Kryven and Wolff (2020)).

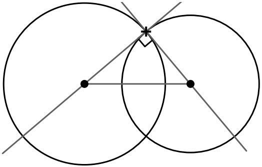


Orthogonal Circles

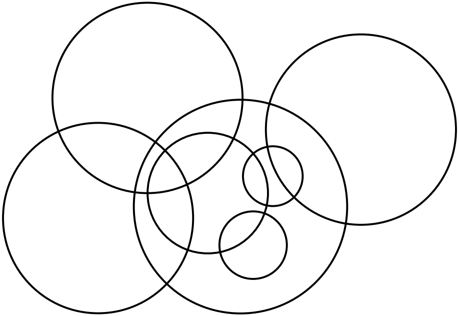
Orthogonal Circles

Definition

Two circles intersect orthogonally if and only if their tangents in their intersection points intersect at a right angle.



Orthogonal Circle Arrangements



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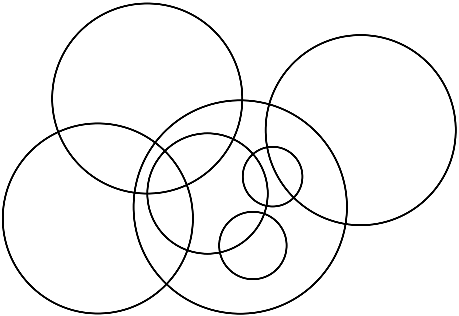
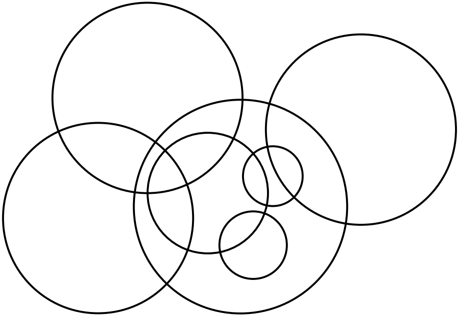


- introduced by Chaplick, Förster, Kryven and Wolff in 2019

Orthogonal Circle Arrangements



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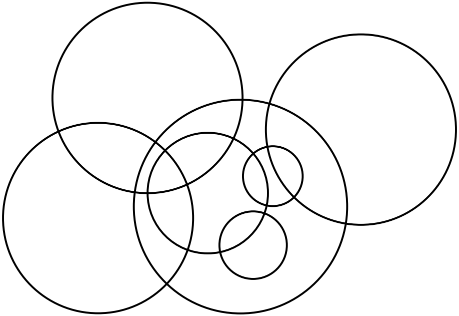


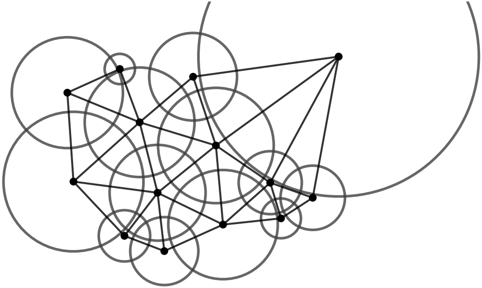
- introduced by Chaplick, Förster, Kryven and Wolff in 2019

Theorem (Chaplick, Förster, Kryven, Wolff (2019))

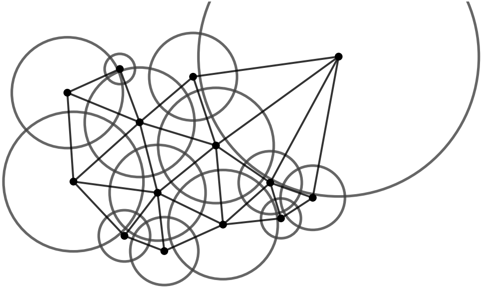
The intersection graph of an arrangement of n orthogonal circles has at most $7n$ edges.

Nonnested Orthogonal Circle Arrangements

Nonnested Orthogonal Circle Arrangements



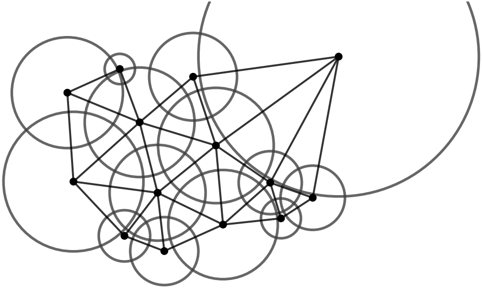
Nonnested Orthogonal Circle Arrangements



Theorem (C, Schulz (2021))

The embedded intersection graph of an arrangement of nonnested orthogonal circles is planar.

Nonnested Orthogonal Circle Arrangements



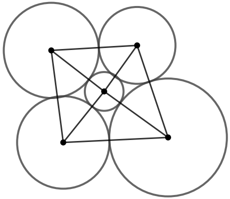
Theorem (C, Schulz (2021))

The embedded intersection graph of an arrangement of nonnested orthogonal circles is planar.

Lemma

No intersection graph of an orthogonal circle arrangement contains a K_4 or an induced C_4.

Circle Arrangements



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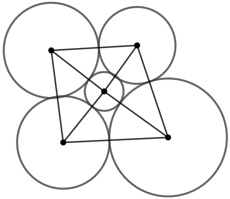

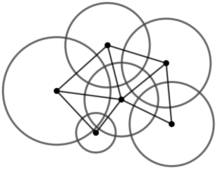

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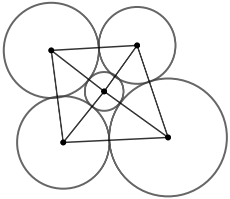

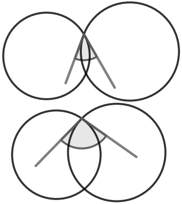

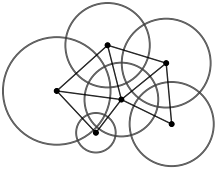

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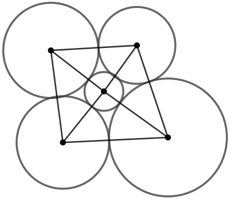

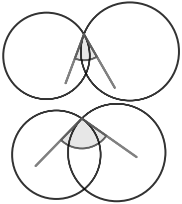

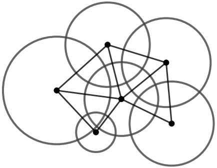

Theorem (C, Schulz (2021))

The embedded intersection graph of an acute nonnested circle arrangement is noncrossing.

General Orthogonal Circle Arrangements

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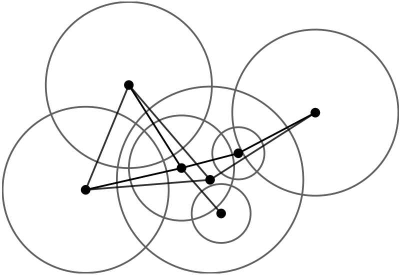

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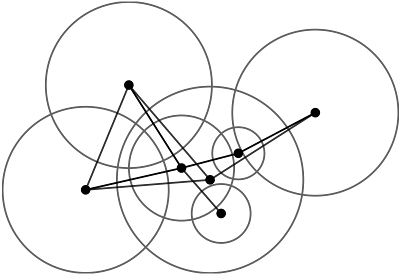
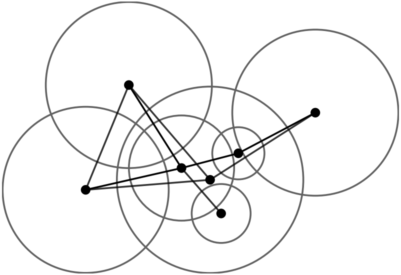


Theorem (Chaplick, Förster, Kryven, Wolff (2019))

For every n, there is an intersection graph of orthogonal circles that contains K_n as a minor.

General Orthogonal Circle Arrangements



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For every n, there is an intersection graph of orthogonal circles that contains K_n as a minor.

Theorem (Chaplick, Förster, Kryven, Wolff (2019))

The intersection graph of an arrangement of n orthogonal circles has at most 7n edges.

General Orthogonal Circle Arrangements

Theorem (C, Schulz (2021))

The intersection graph of an arrangement of n orthogonal circles has at most $5n - 6$ edges.

General Orthogonal Circle Arrangements

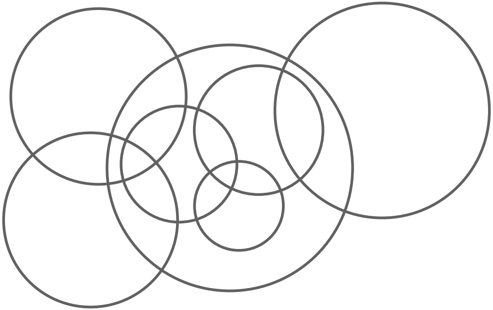
Theorem (C, Schulz (2021))

The intersection graph of an arrangement of n orthogonal circles has at most $5n - 6$ edges.

Lemma

In the intersection graph of an arrangement of n orthogonal circles we can find a subset V that is incident to at most $5n' - 6$ edges, where $n' = |V|$.

General Orthogonal Circle Arrangements



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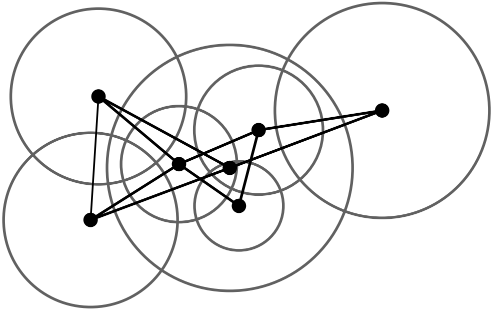

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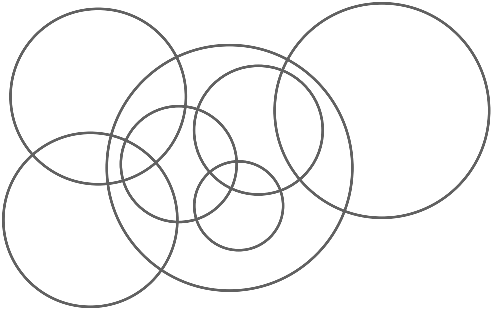

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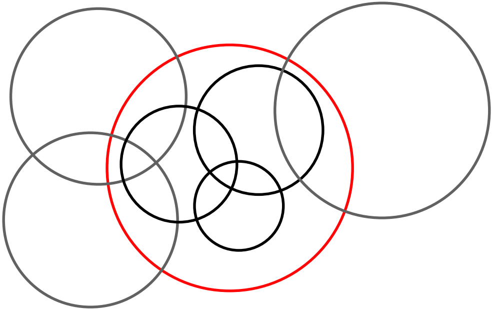
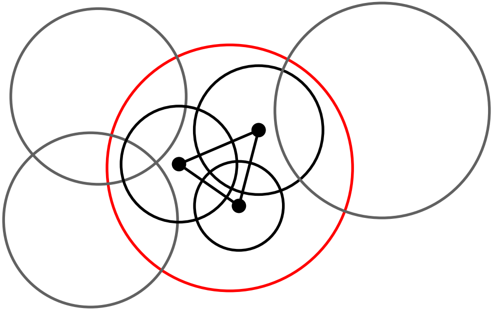


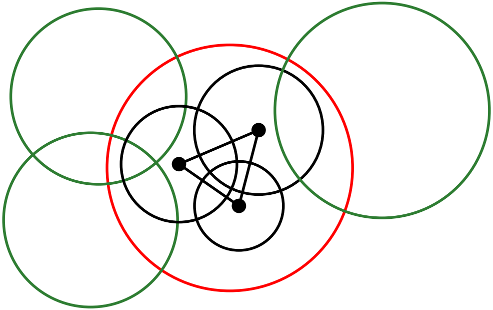
n' black circles

General Orthogonal Circle Arrangements



n' black circles
$3n' - 6$ black edges

General Orthogonal Circle Arrangements



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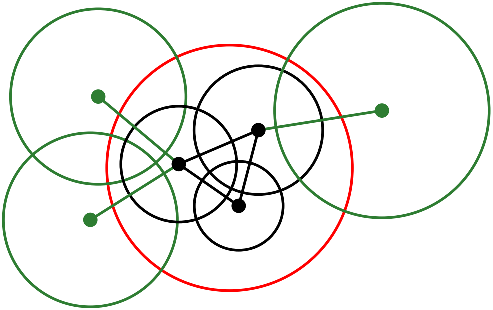
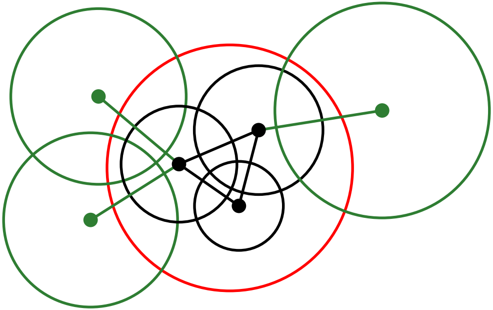
General Orthogonal Circle Arrangements



n' black circles
$3n' - 6$ black edges
$2n'$ green edges

General Orthogonal Circle Arrangements



n' black circles
$3n' - 6$ black edges
$2n'$ green edges
$5n' - 6$ edges incident to vertices of black circles

General Orthogonal Circle Arrangements

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further analysis of the number of green edges leads to

General Orthogonal Circle Arrangements

further analysis of the number of green edges leads to

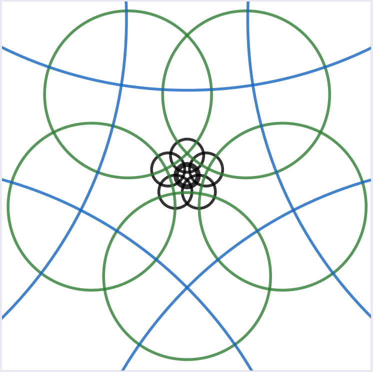
Theorem (C, Schulz (2021))

The intersection graph of an arrangement of n orthogonal circles has at most $\left(4+\frac{5}{11}\right)n$ edges.

Lower bounds

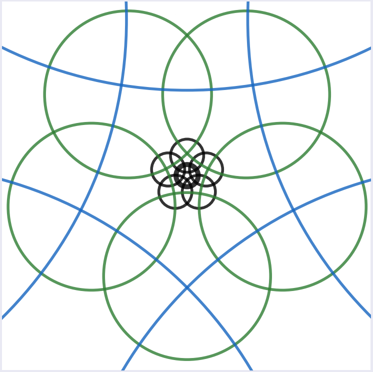
Lower bounds

Nonnested Orthogonal Circle Arrangements



Lower bounds

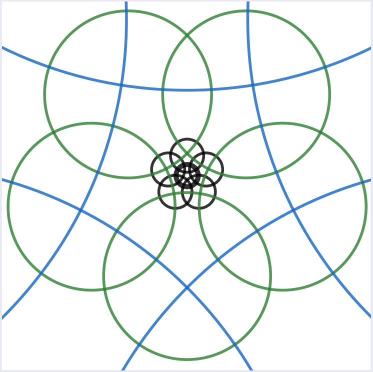
Nonnested Orthogonal Circle Arrangements



$3n - 8$ edges

Lower bounds

Nonnested Orthogonal Circle Arrangements



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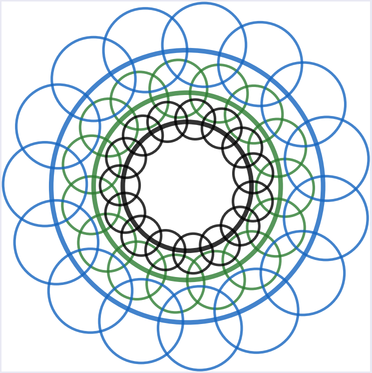
General Orthogonal Circle Arrangements



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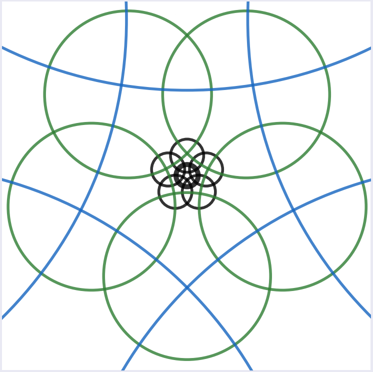

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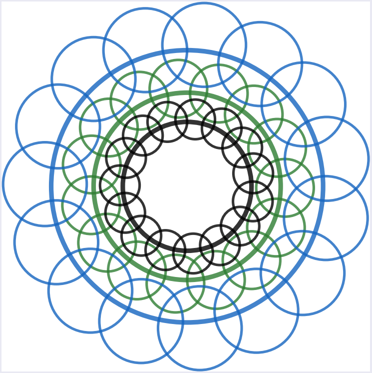
General Orthogonal Circle Arrangements



$4n - O(\sqrt{n})$ edges